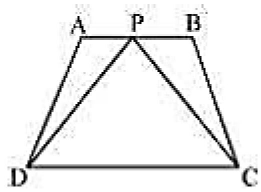
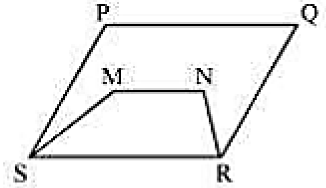
Exercise (9.1)

Question 1:

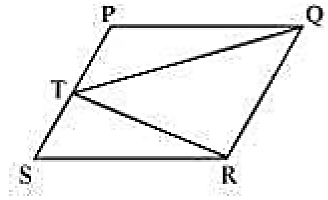
Which of the following figures lie on the same base and between the same parallels. In such a case, write the common base and the two parallels.



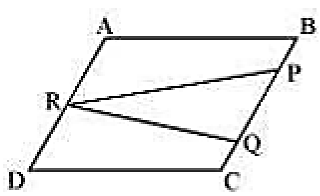
(i)



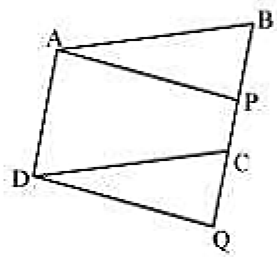
(ii)



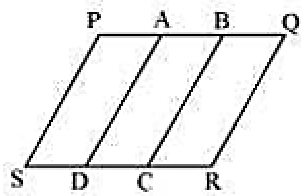
(iii)



(iv)



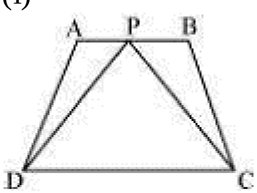
(v)



(vi)

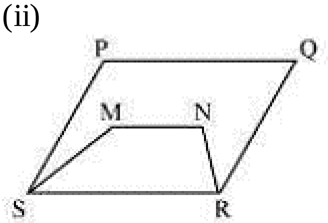
Solution 1:

(i)



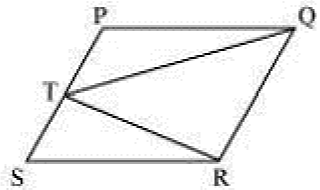
Yes.It can be observed that trapezium ABCD and triangle PCD have a common base CD and these are lying between the same parallel lines AB and CD.

(ii)



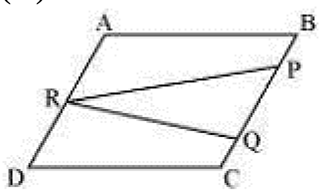
No. It can be observed that parallelogram PQRS and trapezium MNRS have a common base RS. However, their vertices, (i.e., opposite to the common base) P, Q of parallelogram and M, N of trapezium, are not lying on the same line.

(iii)



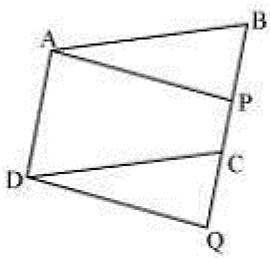
Yes. It can be observed that parallelogram PQRS and triangle TQR have a common base QR and they are lying between the same parallel lines PS and QR.

(iv)



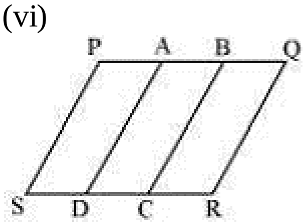
No. It can be observed that parallelogram ABCD and triangle PQR are lying between same parallel lines AD and BC. However, these do not have any common base.

(v)



Yes. It can be observed that parallelogram ABCD and parallelogram APQD have a common base AD and these are lying between the same parallel lines AD and BQ.

(vi)

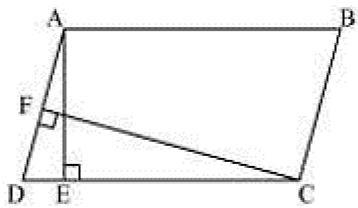


No. It can be observed that parallelogram PBCS and PQRS are lying on the same base PS. However, these do not lie between the same parallel lines.

Exercise (9.2)

Question 1:

In the given figure, ABCD is parallelogram, AE ⊥ DC and CF ⊥ AD. If AB = 16 cm, AE = 8 cm and CF = 10 cm, find AD.



Solution 1:

In parallelogram ABCD, CD = AB = 16 cm

[Opposite sides of a parallelogram are equal]

We know that

Area of a parallelogram = Base × Corresponding altitude

Area of parallelogram ABCD = CD × AE = AD × CF

16 cm × 8 cm = AD × 10 cm

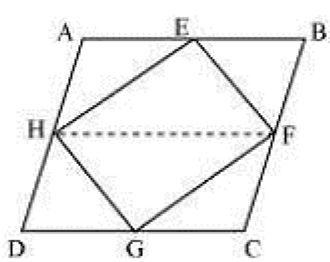
$$AD = \frac{16 \times 8}{10} \text{ cm} = 12.8 \text{ cm}$$

Thus, the length of AD is 12.8 cm.

Question 2:

If E, F, G and H are respectively the mid-points of the sides of a parallelogram ABCD show that ar (EFGH) = $\frac{1}{2}$ ar (ABCD)

Solution 2:



Let us join HF.

In parallelogram ABCD,

AD = BC and AD || BC (Opposite sides of a parallelogram are equal and parallel)

AB = CD (Opposite sides of a parallelogram are equal)

$\Rightarrow \frac{1}{2}AD = \frac{1}{2}BC$ and AH || BF

⇒ AH = BF and AH || BF ($\because$ H and F are the mid-points of AD and BC)

Therefore, ABFH is a parallelogram.

Since ΔHEF and parallelogram ABFH are on the same base HF and between the same parallel lines AB and HF,

$\therefore$ Area (ΔHEF) = $\frac{1}{2}$ Area (ABFH) ... (1)

Similarly, it can be proved that

Area (ΔHGF) = $\frac{1}{2}$ Area (HDCF) ... (2)

On adding Equations (1) and (2), we obtain

Area (ΔHEF) + Area (ΔHGF) = $\frac{1}{2}$ Area (ABFH) + $\frac{1}{2}$ Area (HDCF)

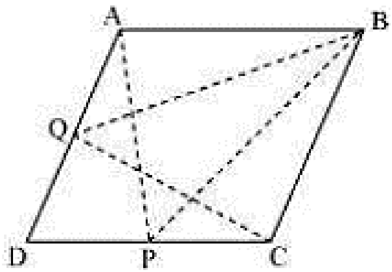
= $\frac{1}{2}$ [Area (ABFH) + Area (HDCF)]

$\Rightarrow$ Area (EFGH) = $\frac{1}{2}$ Area (ABCD)

Question 3:

P and Q are any two points lying on the sides DC and AD respectively of a parallelogram ABCD. Show that ar (APB) = ar (BQC).

Solution 3:



It can be observed that ΔBQC and parallelogram ABCD lie on the same base BC and these are between the same parallel lines AD and BC.

∴ Area (ΔBQC) = $\frac{1}{2}$ Area (ABCD) ... (1)

Similarly, ΔAPB and parallelogram ABCD lie on the same base AB and between the same parallel lines AB and DC.

∴ Area (ΔAPB) = $\frac{1}{2}$ Area (ABCD) ... (2)

From Equations (1) and (2), we obtain

Area (ΔBQC) = Area (ΔAPB)

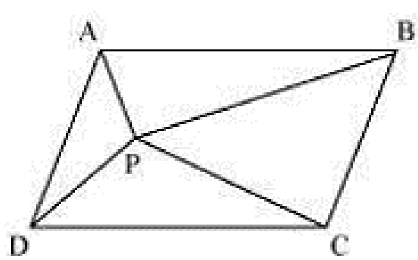
Question 4:

In the given figure, P is a point in the interior of a parallelogram ABCD. Show that

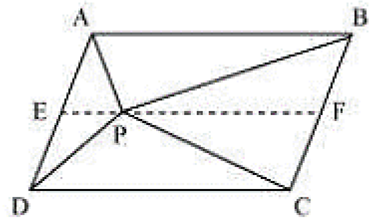
(i) ar (APB) + ar (PCD) = $\frac{1}{2}$ ar (ABCD)

(ii) ar (APD) + ar (PBC) = ar (APB) + ar (PCD)

[*Hint*: Through. P, draw a line parallel to AB]



Solution 4:



(i) Let us draw a line segment EF, passing through point P and parallel to line segment AB.

In parallelogram ABCD,

AB || EF (By construction) ... (1)

ABCD is a parallelogram.

∴ AD || BC (Opposite sides of a parallelogram)

⇒ AE || BF ... (2)

From Equations (1) and (2), we obtain

AB || EF and AE || BF

Therefore, quadrilateral ABFE is a parallelogram.

It can be observed that ΔAPB and parallelogram ABFE are lying on the same base AB and between the same parallel lines AB and EF.

∴ Area (ΔAPB) = $\frac{1}{2}$ Area (ABFE) ... (3)

Similarly, for ΔPCD and parallelogram EFCD,

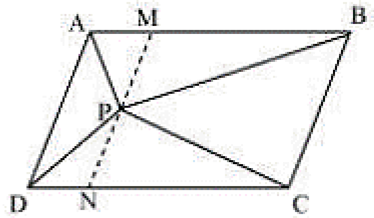
Area (ΔPCD) = $\frac{1}{2}$ Area (EFCD) ... (4)

Adding Equations (3) and (4), we obtain

Area (ΔAPB) + Area (ΔPCD) = $\frac{1}{2}$ [Area (ABFE) + Area (EFCD)]

Area (ΔAPB) + Area (ΔPCD) = $\frac{1}{2}$ Area (ABCD) ... (5)

(ii)



Let us draw a line segment MN, passing through point P and parallel to line segment AD.

In parallelogram ABCD,

MN || AD (By construction) ... (6)

ABCD is a parallelogram.

∴ AB || DC (Opposite sides of a parallelogram)

⇒ AM || DN ... (7)

From Equations (6) and (7), we obtain

MN || AD and AM || DN

Therefore, quadrilateral AMND is a parallelogram.

It can be observed that ΔAPD and parallelogram AMND are lying on the same base AD and between the same parallel lines AD and MN.

$\therefore$ Area (ΔAPD) = $\frac{1}{2}$ Area (AMND) ... (8)

Similarly, for ΔPCB and parallelogram MNCB,

Area (ΔPCB) = $\frac{1}{2}$ Area (MNCB) ... (9)

Adding Equations (8) and (9), we obtain

Area (ΔAPD) + Area (ΔPCB) = $\frac{1}{2}$ [Area (AMND) + Area (MNCB)]

Area (ΔAPD) + Area (ΔPCB) = $\frac{1}{2}$ Area (ABCD) ... (10)

On comparing Equations (5) and (10), we obtain

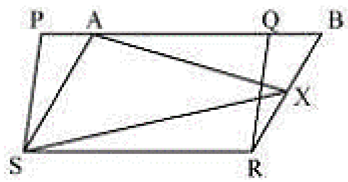
Area (ΔAPD) + Area (ΔPBC) = Area (ΔAPB) + Area (ΔPCD)

Question 5:

In the given figure, PQRS and ABRS are parallelograms and X is any point on side BR. Show that

(i) ar (PQRS) = ar (ABRS)

(ii) ar (ΔPXS) = $\frac{1}{2}$ ar (PQRS)



Solution 5:

(i) It can be observed that parallelogram PQRS and ABRS lie on the same base SR and also, these lie in between the same parallel lines SR and PB.

∴ Area (PQRS) = Area (ABRS) ... (1)

(ii) Consider ΔAXS and parallelogram ABRS.

As these lie on the same base and are between the same parallel lines AS and BR,

∴ Area (ΔAXS) = $\frac{1}{2}$ Area (ABRS) ... (2)

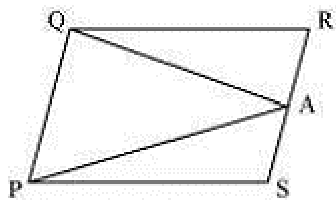
From Equations (1) and (2), we obtain

Area (ΔAXS) = $\frac{1}{2}$ Area (PQRS)

Question 6:

A farmer was having a field in the form of a parallelogram PQRS. She took any point A on RS and joined it to points P and Q. In how many parts the field is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

Solution 6:



From the figure, it can be observed that point A divides the field into three parts.

These parts are triangular in shape − ΔPSA, ΔPAQ, and ΔQRA

Area of ΔPSA + Area of ΔPAQ + Area of ΔQRA = Area of parallelogram PQRS ... (1)

We know that if a parallelogram and a triangle are on the same base and between the same parallels, then the area of the triangle is half the area of the parallelogram.

$\therefore$ Area (ΔPAQ) = $\frac{1}{2}$ Area (PQRS) ... (2)

From Equations (1) and (2), we obtain

Area (ΔPSA) + Area (ΔQRA) = $\frac{1}{2}$ Area (PQRS) ... (3)

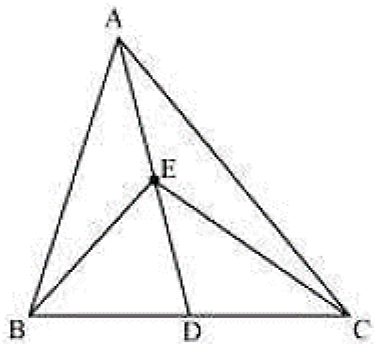
Clearly, it can be observed that the farmer must sow wheat in triangular part PAQ and pulses in other two triangular parts PSA and QRA or wheat in triangular parts PSA and QRA and pulses in triangular parts PAQ.

Exercise (9.3)

Question 1:

In the given figure, E is any point on median AD of a ΔABC. Show that

ar (ABE) = ar (ACE)



Solution 1:

AD is the median of ΔABC. Therefore, it will divide ΔABC into two triangles of equal areas.

∴ Area (ΔABD) = Area (ΔACD) ... (1)

ED is the median of ΔEBC.

∴ Area (ΔEBD) = Area (ΔECD) ... (2)

On subtracting Equation (2) from Equation (1), we obtain

Area (ΔABD) − Area (EBD) = Area (ΔACD) − Area (ΔECD)

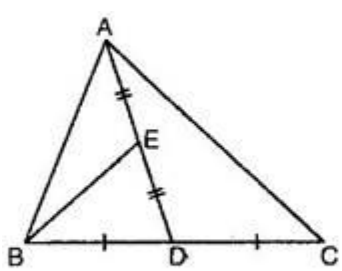
Area (ΔABE) = Area (ΔACE)

Question 2: In a triangle ABC, E is the mid-point of median AD.

Show that ar(Δ BED) =1/4 ar(ΔABC) .

Solution 2 :

Given: A ΔABC, AD is the median and E is the mid-point of median AD.



To prove: ar(ΔBED) = 1/4 ar(ΔABC)

Proof : In ΔABC, AD is the median.

∴ ar (ΔABD) = ar (ΔADC)

[∴ Median divides a Δ into two Δs of equal area]

ar(ΔABD) = $\frac{1}{2}$ ar(ABC) …..(i)

In ΔABD, BE is the median.

ar (ΔBED) = ar (ΔBAE)

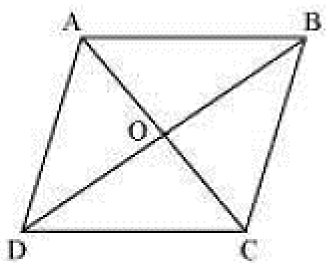
∴ ar (ΔBED)= $\frac{1}{2}$ ar(ΔABD)

= ar (ΔBED) = $\frac{1}{2}$ [$\frac{1}{2}$ ar (ΔABC)] = $\frac{1}{2}$ ar (ΔABC)

Question 3:

Show that the diagonals of a parallelogram divide it into four triangles of equal area.

Solution 3:



We know that diagonals of parallelogram bisect each other.

Therefore, O is the mid-point of AC and BD.

BO is the median in ΔABC. Therefore, it will divide it into two triangles of equal areas.

∴ Area (ΔAOB) = Area (ΔBOC) ... (1)

In ΔBCD, CO is the median.

∴ Area (ΔBOC) = Area (ΔCOD) ... (2)

Similarly, Area (ΔCOD) = Area (ΔAOD) ... (3)

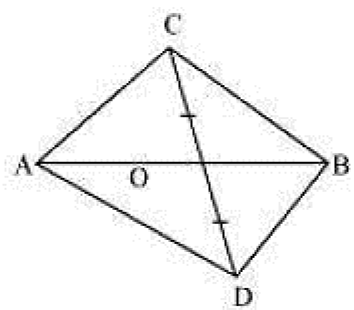
From Equations (1), (2), and (3), we obtain

Area (ΔAOB) = Area (ΔBOC) = Area (ΔCOD) = Area (ΔAOD)

Therefore, it is evident that the diagonals of a parallelogram divide it into four triangles of equal area.

Question 4:

In the given figure, ABC and ABD are two triangles on the same base AB. If line-segment CD is bisected by AB at O, show that ar (ABC) = ar (ABD).



Solution 4:

Consider ΔACD.

Line-segment CD is bisected by AB at O. Therefore, AO is the median of ΔACD.

∴ Area (ΔACO) = Area (ΔADO) ... (1)

Considering ΔBCD, BO is the median.

∴ Area (ΔBCO) = Area (ΔBDO) ... (2)

Adding Equations (1) and (2), we obtain

Area (ΔACO) + Area (ΔBCO) = Area (ΔADO) + Area (ΔBDO)

⇒ Area (ΔABC) = Area (ΔABD)

Question 5 :

D, E and F are respectively the mid-points of the sides BC, CA and AB of a ΔABC. Show that:

(i) BDEF is a parallelogram.

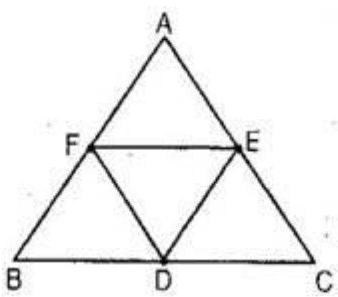
(ii) ar (DEF) = $\frac{1}{4}$ar (ABC)

(iii) ar (BDEF) = $\frac{1}{2}$ar (ABC)

Solution 5:

(i) F is the mid-point of AB and E is the mid-point of AC.

∴ FE ∥ BC and FE = $\frac{1}{2}$ BD



Line joining the mid-points of two sides of a triangle is parallel to the third and half of It

∴FE ∥ BD [BD is the part of BC]

And FE = BD

Also, D is the mid-point of BC.

∴ BD = $\frac{1}{2}$ BC

And FE ∥ BC and FE = BD

Again E is the mid-point of AC and D is the mid-point of BC.

$\therefore$ DE $\parallel$ AB and DE = $\frac{1}{2}$ AB

DE $\parallel$ AB [BF is the part of AB]

And DE = BF

Again F is the mid-point of AB.

$\therefore$ BF = $\frac{1}{2}$ AB

But DE = $\frac{1}{2}$ AB

$\therefore$ DE = BF

Now we have FE $\parallel$ BD and DE $\parallel$ BF

And FE = BD and DE = BF

Therefore, BDEF is a parallelogram.

(ii) BDEF is a parallelogram.

$\therefore$ ar(ΔBDF) = ar(ΔDEF) ………..(i)

[diagonals of parallelogram divides it in two triangles of equal area]

DCEF is also parallelogram.

$\therefore$ ar (Δ DEF) = ar (ΔDEC) ……….(ii)

Also, AEDF is also parallelogram.

$\therefore$ ar (ΔAFE) = ar (ΔDEF) ……….(iii)

From eq. (i), (ii) and (iii),

ar (ΔDEF) = ar (ΔBDF) = ar (ΔDEC) = ar (ΔAFE) ……….(iv)

Now, ar (ΔABC) = ar (ΔDEF) + ar (ΔBDF) + ar (ΔDEC) + ar (ΔAFE) ……….(v)

ar (ΔABC) = ar (ΔDEF) + ar (ΔDEF) + ar (ΔDEF) + ar (ΔDEF)

[Using (iv) & (v)]

ar (ΔABC) = 4 × ar (ΔDEF)

ar (ΔDEF) = $\frac{1}{4}$ ar (ΔABC)

(iii) ar (∥gm BDEF) = ar (ΔBDF) + ar (ΔDEF) = ar (ΔDEF) + ar (ΔDEF) [Using (iv)]

ar (∥gm BDEF) = 2 ar (ΔDEF)

ar (∥gm BDEF) = $2\times\frac{1}{4}$ ar (ΔABC)

ar (∥gm BDEF) = $\frac{1}{2}$ ar (ΔABC)

Question 6:

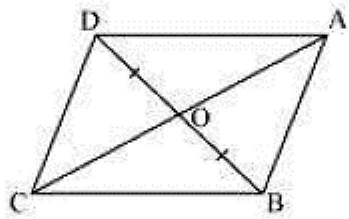
In the given figure, diagonals AC and BD of quadrilateral ABCD intersect at O such that OB = OD. If AB = CD, then show that:

(i) ar (DOC) = ar (AOB)

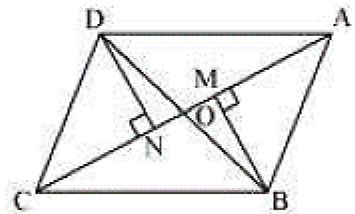
(ii) ar (DCB) = ar (ACB)

(iii) DA || CB or ABCD is a parallelogram.

[*Hint*: From D and B, draw perpendiculars to AC.]



Solution 6:



Let us draw DN ⊥ AC and BM ⊥ AC.

(i) In ΔDON and ΔBOM,

∠DNO = ∠BMO (By construction)

∠DON = ∠BOM (Vertically opposite angles)

OD = OB (Given)

By AAS congruence rule,

ΔDON ≅ ΔBOM

DN = BM ... (1)

We know that congruent triangles have equal areas.

Area (ΔDON) = Area (ΔBOM) ... (2)

In ΔDNC and ΔBMA,

∠DNC = ∠BMA (By construction)

CD = AB (given)

DN = BM [Using Equation (1)]

∴ ΔDNC ≅ ΔBMA (RHS congruence rule)

∴ Area (ΔDNC) = Area (ΔBMA) ... (3)

On adding Equations (2) and (3), we obtain

Area (ΔDON) + Area (ΔDNC) = Area (ΔBOM) + Area (ΔBMA)

Therefore, Area (ΔDOC) = Area (ΔAOB)

(ii) We obtained,

Area (ΔDOC) = Area (ΔAOB)

∴ Area (ΔDOC) + Area (ΔOCB) = Area (ΔAOB) + Area (ΔOCB)

(Adding Area (ΔOCB) to both sides)

∴ Area (ΔDCB) = Area (ΔACB)

(iii) We obtained,

Area (ΔDCB) = Area (ΔACB)

If two triangles have the same base and equal areas, then these will lie between the same parallels.

∴ DA || CB ... (4)

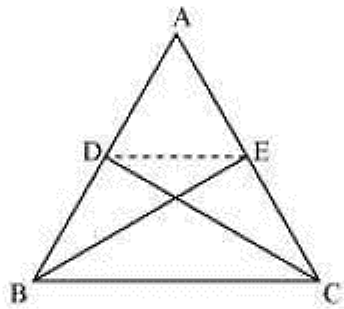
In quadrilateral ABCD, one pair of opposite sides is equal (AB = CD) and the other pair of opposite sides is parallel (DA || CB).

Therefore, ABCD is a parallelogram.

Question 7:

D and E are points on sides AB and AC respectively of ΔABC such that ar (DBC) = ar (EBC). Prove that DE || BC.

Solution 7:



Since ΔBCE and ΔBCD are lying on a common base BC and also have equal areas, ΔBCE and ΔBCD will lie between the same parallel lines.

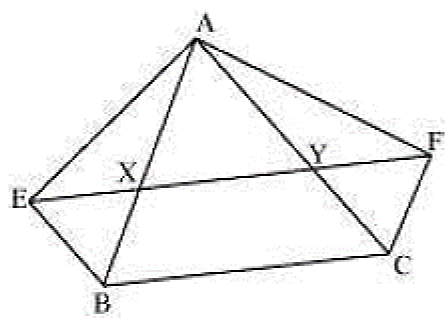
∴ DE || BC

Question 8:

XY is a line parallel to side BC of a triangle ABC. If BE || AC and CF || AB meet XY at E and F respectively, show that

ar (ABE) = ar (ACF)

Solution 8:



It is given that

XY || BC = EY || BC

BE || AC = BE || CY

Therefore, EBCY is a parallelogram.

It is given that

XY || BC = XF || BC

FC || AB = FC || XB

Therefore, BCFX is a parallelogram.

Parallelograms EBCY and BCFX are on the same base BC and between the same parallels BC and EF.

∴ Area (EBCY) = $\frac{1}{2}$ Area (BCFX) ... (1)

Consider parallelogram EBCY and ΔAEB

These lie on the same base BE and are between the same parallels BE and AC.

∴ Area (ΔABE) = $\frac{1}{2}$ Area (EBCY) ... (2)

Also, parallelogram ΔCFX and ΔACF are on the same base CF and between the same parallels CF and AB.

∴ Area (ΔACF) = $\frac{1}{2}$ Area (BCFX) ... (3)

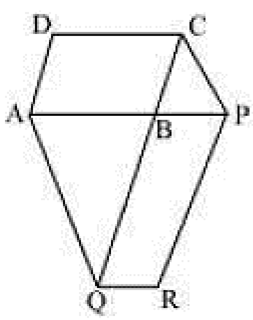
From Equations (1), (2), and (3), we obtain

Area (ΔABE) = Area (ΔACF)

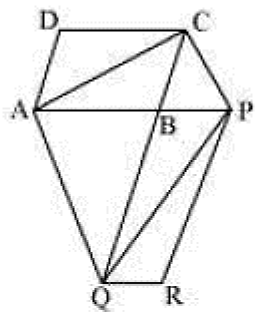
Question 9:

The side AB of a parallelogram ABCD is produced to any point P. A line through A and parallel to CP meets CB produced at Q and then parallelogram PBQR is completed (see the following figure). Show that ar (ABCD) = ar (PBQR).

[*Hint*: Join AC and PQ. Now compare area (ACQ) and area (APQ)]



Solution 9:



Let us join AC and PQ.

ΔACQ and ΔAQP are on the same base AQ and between the same parallels AQ and CP.

Area (ΔACQ) = Area (ΔAPQ)

Area (ΔACQ) − Area (ΔABQ) = Area (ΔAPQ) − Area (ΔABQ)

Area (ΔABC) = Area (ΔQBP) ... (1)

Since AC and PQ are diagonals of parallelograms ABCD and PBQR respectively,

Area (ΔABC) = $\frac{1}{2}$ Area (ABCD) ... (2)

Area (ΔQBP) = $\frac{1}{2}$ Area (PBQR) ... (3)

From Equations (1), (2), and (3), we obtain

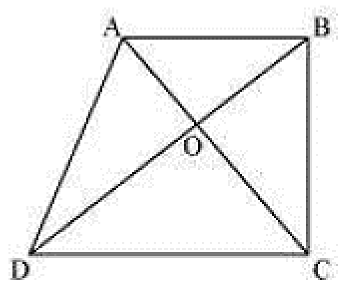
$\frac{1}{2}$ Area (ABCD) = $\frac{1}{2}$ Area (PBQR)

Area (ABCD) = Area (PBQR)

Question 10:

Diagonals AC and BD of a trapezium ABCD with AB || DC intersect each other at O. Prove that ar (AOD) = ar (BOC).

Solution 10:



It can be observed that ΔDAC and ΔDBC lie on the same base DC and between the same parallels AB and CD.

Area (ΔDAC) = Area (ΔDBC)

Area (ΔDAC) − Area (ΔDOC) = Area (ΔDBC) − Area (ΔDOC)

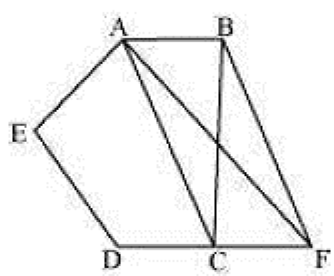
Area (ΔAOD) = Area (ΔBOC)

Question 11:

In the given figure, ABCDE is a pentagon. A line through B parallel to AC meets DC produced at F. Show that

(i) ar (ACB) = ar (ACF)

(ii) ar (AEDF) = ar (ABCDE)



Solution 11:

(i) ΔACB and ΔACF lie on the same base AC and are between

The same parallels AC and BF.

Area (ΔACB) = Area (ΔACF)

(ii) It can be observed that

Area (ΔACB) = Area (ΔACF)

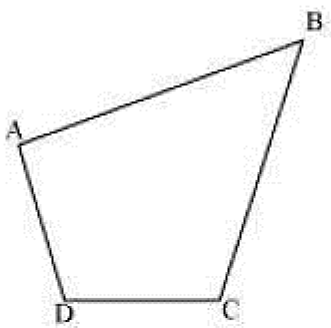
Area (ΔACB) + Area (ACDE) = Area (ΔACF) + Area (ACDE)

Area (ABCDE) = Area (AEDF)

Question 12:

A villager Itwaari has a plot of land of the shape of a quadrilateral. The Gram Panchayat of the village decided to take over some portion of his plot from one of the corners to construct a Health Centre. Itwaari agrees to the above proposal with the condition that he should be given equal amount of land in lieu of his land adjoining his plot so as to form a triangular plot. Explain how this proposal will be implemented.

Solution 12:



Let quadrilateral ABCD be the original shape of the field.

The proposal may be implemented as follows.

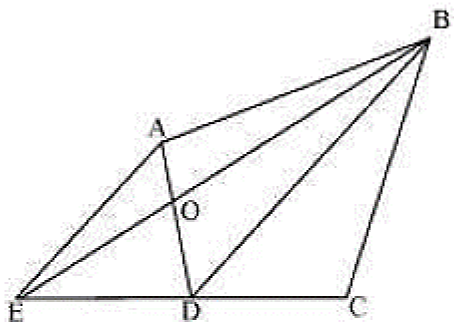
Join diagonal BD and draw a line parallel to BD through point A.

Let it meet the extended side CD of ABCD at point E.

Join BE and AD. Let them intersect each other at O.

Then, portion ΔAOB can be cut from the original field so that the new shape of the field will be ΔBCE. (See figure).

We have to prove that the area of ΔAOB (portion that was cut so as to construct Health Centre) is equal to the area of ΔDEO (portion added to the field so as to make the area of the new field so formed equal to the area of the original field).



It can be observed that ΔDEB and ΔDAB lie on the same base BD and are between the same parallels BD and AE.

Area (ΔDEB) = Area (ΔDAB)

Area (ΔDEB) − Area (ΔDOB) = Area (ΔDAB) − Area (ΔDOB)

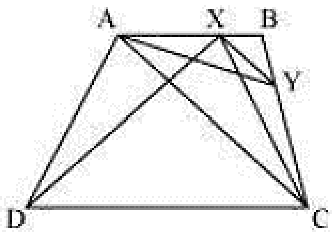
Area (ΔDEO) = Area (ΔAOB)

Question 13:

ABCD is a trapezium with AB || DC. A line parallel to AC intersects AB at X and BC at Y. Prove that ar (ADX) = ar (ACY).

[*Hint*: Join CX.]

Solution 13:



It can be observed that ΔADX and ΔACX lie on the same base AX and are between the same parallels AB and DC.

Area (ΔADX) = Area (ΔACX) ... (1)

ΔACY and ΔACX lie on the same base AC and are between the same parallels AC and XY.

Area (ΔACY) = Area (ACX) ... (2)

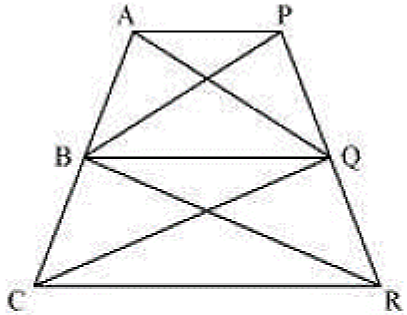
From Equations (1) and (2), we obtain

Area (ΔADX) = Area (ΔACY)

Question 14:

In the given figure, AP || BQ || CR. Prove that ar (AQC) = ar (PBR).

Solution 14:



Since ΔABQ and ΔPBQ lie on the same base BQ and are between the same parallels AP and BQ,

∴ Area (ΔABQ) = Area (ΔPBQ) ... (1)

Again, ΔBCQ and ΔBRQ lie on the same base BQ and are between the same parallels BQ and CR.

∴ Area (ΔBCQ) = Area (ΔBRQ) ... (2)

On adding Equations (1) and (2), we obtain

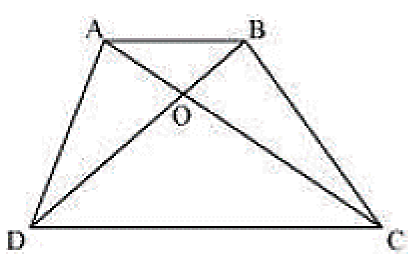
Area (ΔABQ) + Area (ΔBCQ) = Area (ΔPBQ) + Area (ΔBRQ)

∴ Area (ΔAQC) = Area (ΔPBR)

Question 15:

Diagonals AC and BD of a quadrilateral ABCD intersect at O in such a way that ar (AOD) = ar (BOC). Prove that ABCD is a trapezium.

Solution 15:



It is given that

Area (ΔAOD) = Area (ΔBOC)

Area (ΔAOD) + Area (ΔAOB) = Area (ΔBOC) + Area (ΔAOB)

Area (ΔADB) = Area (ΔACB)

We know that triangles on the same base having areas equal to each other lie between the same parallels.

Therefore, these triangles, ΔADB and ΔACB, are lying between the same parallels.

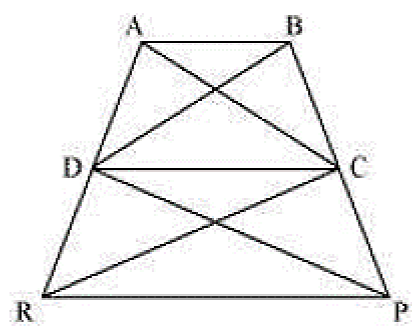
i.e., AB || CD

Therefore, ABCD is a trapezium.

Question 16:

In the given figure, ar (DRC) = ar (DPC) and ar (BDP) = ar (ARC). Show that both the quadrilaterals ABCD and DCPR are trapeziums.

Solution 16:



It is given that

Area (ΔDRC) = Area (ΔDPC)

As ΔDRC and ΔDPC lie on the same base DC and have equal areas, therefore, they must lie between the same parallel lines.

∴ DC || RP

Therefore, DCPR is a trapezium.

It is also given that

Area (ΔBDP) = Area (ΔARC)

Area (ΔBDP) − Area (ΔDPC) = Area (ΔARC) − Area (ΔDRC)

∴ Area (ΔBDC) = Area (ΔADC)

Since ΔBDC and ΔADC are on the same base CD and have equal areas, they must lie between the same parallel lines.

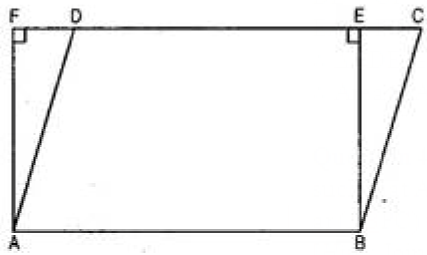
∴ AB || CD

Therefore, ABCD is a trapezium.

Exercise (9.4)

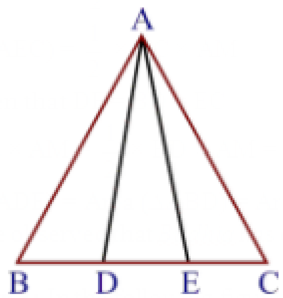
Question 1:
Parallelogram ABCD and rectangle ABEF are on the same base AB and have equal areas. Show that the perimeter of the parallelogram is greater than that of the rectangle.

Solution 1: As the parallelogram and the rectangle have the same base and equal area, therefore, these will also lie between the same parallels.
Consider the parallelogram ABCD and rectangle ABEF as follows.



Here, it can be observed that parallelogram ABCD and rectangle ABEF are between the same parallels AB and CF.
We know that opposite sides of a parallelogram or a rectangle are of equal lengths.
Therefore,
AB = EF (For rectangle)
AB = CD (For parallelogram)
∴ CD = EF
∴ AB + CD = AB + EF ... (1)
Of all the line segments that can be drawn to a given line from a point not lying on it, the perpendicular line segment is the shortest.
∴ AF < AD
And similarly, BE < BC
∴ AF + BE < AD + BC ... (2)
From Equations (1) and (2), we obtain
AB + EF + AF + BE < AD + BC + AB + CD
Perimeter of rectangle ABEF < Perimeter of parallelogram ABCD.

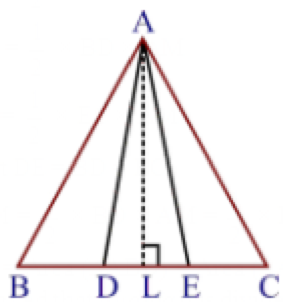
Question 2:
In the following figure, D and E are two points on BC such that BD = DE = EC. Show that ar (ABD) = ar (ADE) = ar (AEC).



Can you now answer the question that you have left in the 'Introduction' of this chapter, whether the field of Budhia has been actually divided into three parts of equal area?

[*Remark*: Note that by taking BD = DE = EC, the triangle ABC is divided into three triangles ABD, ADE and AEC of equal areas. In the same way, by dividing BC into n equal parts and joining the points of division so obtained to the opposite vertex of BC, you can divide ΔABC into *n* triangles of equal areas.]

Solution 2:
Let us draw a line segment AL ⊥ BC.



We know that,

Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Altitude}$

Area (ΔADE) = $\frac{1}{2} \times DE \times AL$

Area (ΔABD) = $\frac{1}{2} \times BD \times AL$

Area (ΔAEC) = $\frac{1}{2} \times EC \times AL$

It is given that DE = BD = EC

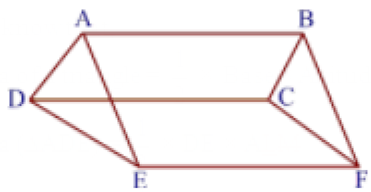
$\frac{1}{2} \times DE \times AL = \frac{1}{2} \times BD \times AL = \frac{1}{2} \times EC \times AL$

Area (ΔADE) = Area (ΔABD) = Area (ΔAEC)

It can be observed that *Budhia* has divided her field into 3 equal parts.

Question 3:

In the following figure, ABCD, DCFE and ABFE are parallelograms.
Show that ar (ΔADE) = ar (ΔBCF).



Solution 3:

It is given that ABCD is a parallelogram. We know that opposite sides of a parallelogram are equal.

∴ AD = BC ... (1)

Similarly, for parallelograms DCEF and ABFE, it can be proved that

DE = CF ... (2)

And, EA = FB ... (3)

In ΔADE and ΔBCF,

AD = BC [Using equation (1)]

DE = CF [Using equation (2)]

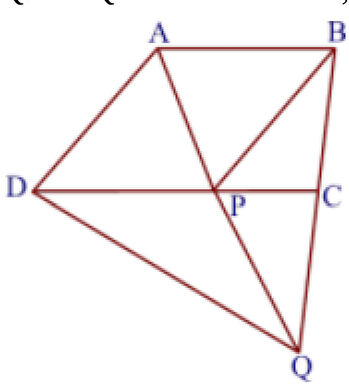
EA = FB [Using equation (3)]

∴ ΔADE ≅ ΔBCF (SSS congruence rule)

∴ Area (ΔADE) = Area (ΔBCF)

Question 4:

In the following figure, ABCD is parallelogram and BC is produced to a point Q such that AD = CQ. If AQ intersect DC at P, show that ar (ΔBPC) = ar (ΔDPQ).

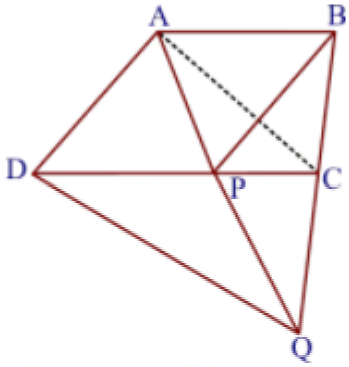


[*Hint:* Join AC.]

Solution 4:

It is given that ABCD is a parallelogram.

AD || BC and AB || DC(Opposite sides of a parallelogram are parallel to each other)
Join point A to point C.



Consider ΔAPC and ΔBPC

ΔAPC and ΔBPC are lying on the same base PC and between the same parallels PC and AB. Therefore,

Area (ΔAPC) = Area (ΔBPC) ... (1)

In quadrilateral ACDQ, it is given that
AD = CQ

Since ABCD is a parallelogram,

AD || BC (Opposite sides of a parallelogram are parallel)
CQ is a line segment which is obtained when line segment BC is produced.
∴ AD || CQ
We have,
AC = DQ and AC || DQ
Hence, ACQD is a parallelogram.

Consider BDCQ and BACQ
These are on the same base CQ and between the same parallels CQ and AD.

Therefore,
Area (ΔDCQ) = Area (ΔACQ)
∴ Area (ΔDCQ) − Area (ΔPQC) = Area (ΔACQ) − Area (ΔPQC)
∴ Area (ΔDPQ) = Area (ΔAPC) ... (2)
From equations (1) and (2), we obtain

Area (ΔBPC) = Area (ΔDPQ)

Question 5:

In the following figure, ABC and BDE are two equilateral triangles such that D is the mid-point of BC. If AE intersects BC at F, show that

(i) ar (BDE) = $\frac{1}{4}$ ar (ABC)

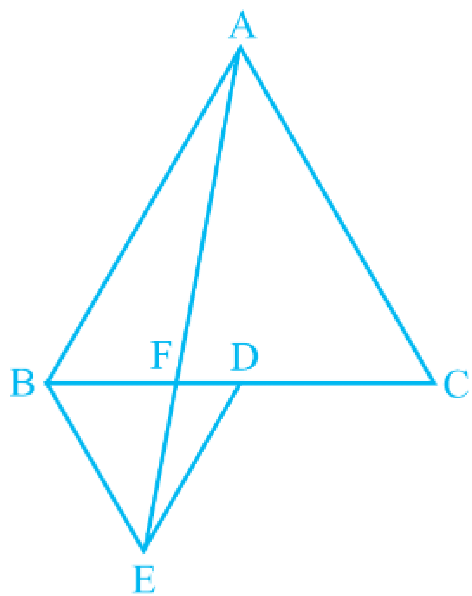
(ii) ar (BDE) = $\frac{1}{2}$ ar (BAE)

(iii) ar (ABC) = 2 ar (BEC)

(iv) ar (BFE) = ar (AFD)

(v) ar (BFE) = 2 ar (FED)

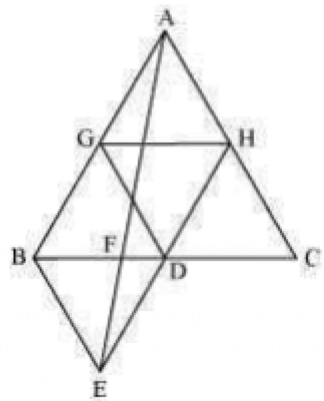
(vi) ar (FED) = $\frac{1}{8}$ ar (AFC)



[Hint: Join EC and AD. Show that BE || AC and DE || AB etc.]

Solution 5:

(i) Let G and H be the mid-points of side AB and AC respectively.
Line segment GH is joining the mid-points and is parallel to third side.
Therefore, BC will be half of the length of BC (mid-point theorem).



∴ GH = $\frac{1}{2}$ BC and GH || BD

∴ GH = BD = DC and GH || BD (D is the mid-point of BC)

Similarly,

- GD = HC = HA
- HD = AG = BG

  Therefore, clearly ΔABC is divided into 4 equal equilateral triangles viz ΔBGD, ΔAGH, ΔDHC and ΔGHD

  In other words, ΔBGD = $\frac{1}{4}$ ΔABC

  Now consider ΔBDG and ΔBDE

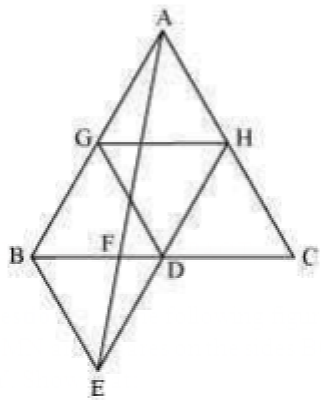
  BD = BD (Common base)
  As both triangles are equilateral triangle, we can say
  BG = BE
  DG = DE

  Therefore, ΔBDG $\cong$ ΔBDE [By SSS congruency]

Thus, area (ΔBDG) = area (ΔBDE)

ar (ΔBDE) = $\frac{1}{4}$ ar (ΔABC)

Hence proved



(ii) Area (ΔBDE) = Area (ΔAED) (Common base DE and DE||AB)
Area (ΔBDE) − Area (ΔFED) = Area (ΔAED) − Area (ΔFED)
Area (ΔBEF) = Area (ΔAFD) … (1)

Now, Area (ΔABD) = Area (ΔABF) + Area (ΔAFD)
Area (ΔABD) = Area (ΔABF) + Area (ΔBEF) [From equation (1)]
Area (ΔABD) = Area (ΔABE) ... (2)
AD is the median in ΔABC.

$$\text{ar}(\Delta ABD) = \frac{1}{2}\text{ar}(\Delta ABC)$$

$$= \frac{4}{2}\text{ar}(\Delta BDE) \quad \text{(As proved earlier)}$$

$$\text{ar}(\Delta ABD) = 2\,\text{ar}(\Delta BDE) \quad (3)$$

From (2) and (3), we obtain
2 ar (ΔBDE) = ar (ΔABE)

$$\text{ar (BDE)} = \frac{1}{2}\ \text{ar (BAE)}$$

(iii)

ar (ΔABE) = ar (ΔBEC) (Common base BE and BE||AC)
ar (ΔABF) + ar (ΔBEF) = ar (ΔBEC)

Using equation (1), we obtain
ar (ΔABF) + ar (ΔAFD) = ar (ΔBEC)
ar (ΔABD) = ar (ΔBEC)

$$\frac{1}{2}\text{ar}(\Delta ABC) = \text{ar}(\Delta BEC)$$

ar (ΔABC) = 2 ar (ΔBEC)

(iv) It is seen that ΔBDE and ar ΔAED lie on the same base (DE) and between the parallels DE and AB.
∴ ar (ΔBDE) = ar (ΔAED)
∴ ar (ΔBDE) − ar (ΔFED) = ar (ΔAED) − ar (ΔFED)
∴ ar (ΔBFE) = ar (ΔAFD)

(v) Let h be the height of vertex E, corresponding to the side BD in ΔBDE.
Let H be the height of vertex A, corresponding to the side BC in ΔABC.

In (i), it was shown that ar (BDE) = $\frac{1}{4}$ ar (ABC)

In (iv), it was shown that ar (ΔBFE) = ar (ΔAFD).
∴ ar (ΔBFE) = ar (ΔAFD)
= 2 ar (ΔFED)
Hence,

(vi)

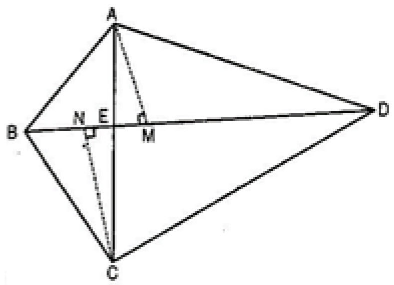
ar (ΔAFC) = ar (ΔAFD) + ar (ΔADC) = 2 ar (ΔFED) + $\frac{1}{2}$ ar (ΔABC) [using (v)

= 2 ar (ΔFED) + $\frac{1}{2}$ [4 × ar (ΔBDE)] [Using result of part (i)]
= 2 ar (ΔFED) + 2 ar (ΔBDE) = 2 ar (ΔFED) + 2 ar (ΔAED)
[ΔBDE and ΔAED are on the same base and between same parallels]
= 2 ar (ΔFED) + 2 [ar (ΔAFD) + ar (ΔFED)]
= 2 ar (ΔFED) + 2 ar (ΔAFD) + 2 ar (ΔFED) [Using (viii)]
= 4 ar (ΔFED) + 4 ar (ΔFED)

⇒ ar (ΔAFC) = 8 ar (ΔFED)
⇒ ar (ΔFED) = $\frac{1}{8}$ ar (ΔAFC)

Question 6:
Diagonals AC and BD of a quadrilateral ABCD intersect each other at P. Show that
[Hint: From A and C, draw perpendiculars to BD]

Solution 6:
Given: A quadrilateral ABCD, in which diagonals AC and BD intersect each other at point E.



To Prove : $ar(\Delta AED) \times ar(\Delta BEC)$
$= ar(\Delta ABE) \times ar(\Delta CDE)$

Construction: From A, draw $AM \perp BD$ AM BD and from C, draw $CN \perp BD$.

$$\text{Proof : } ar(\Delta ABE) = \frac{1}{2} \times BE \times AM ....(i)$$

$$ar(\Delta AED) = \frac{1}{2} \times DE \times AM ....(ii)$$

Dividing eq. (ii) by (i), we get,

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{\frac{1}{2} \times DE \times AM}{\frac{1}{2} \times BE \times AM}$$

$$\Rightarrow \frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{DE}{BE} ....(iii)$$

$$\text{Similarly } \frac{ar(\Delta CDE)}{ar(\Delta BEC)} = \frac{DE}{BE} ....(iv)$$

From eq. (iii) and (iv), we get

$$\frac{ar(\Delta AED)}{ar(\Delta ABE)} = \frac{ar(\Delta CDE)}{ar(\Delta BEC)}$$

$$\Rightarrow ar(\Delta AED) \times ar(\Delta BEC) = ar(\Delta ABE) \times ar(\Delta CDE)$$

Hence proved.

Question 7:

P and Q are respectively the mid-points of sides AB and BC of a triangle ABC and R is the mid-point of AP, show that

(i) ar (PRQ) = $\frac{1}{2}$ ar (ARC)

(ii) ar (RQC) = $\frac{3}{8}$ ar (ABC)

(iii) ar (PBQ) = ar (ARC)

Solution 7:

(i) PC is the median of Δ ABC.

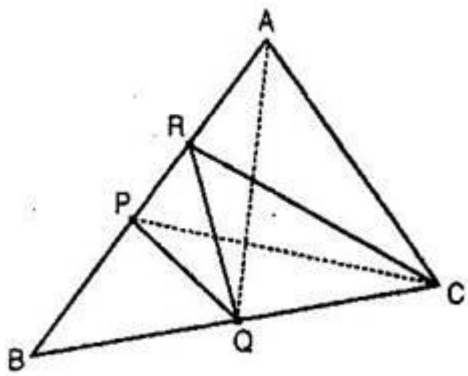
$\therefore$ ar (Δ BPC) = ar (Δ APC) ……….(i)

RC is the median Δ of APC.

$\therefore$ ar (Δ ARC) = $\frac{1}{2}$ ar (Δ APC) ……….(ii)

[Median divides the triangle into two triangles of equal area]

PQ is the median of Δ BPC.



$\therefore$ ar (Δ PQC) = $\frac{1}{2}$ ar (Δ BPC) ……….(iii)

From eq. (i) and (iii), we get,

ar (Δ PQC) = $\frac{1}{2}$ ar (Δ APC) ……….(iv)

From eq. (ii) and (iv), we get,

ar (Δ PQC) = ar (Δ ARC) ……….(v)

We are given that P and Q are the mid-points of AB and BC respectively.

$\therefore$ PQ $\parallel$ AC and PA = $\frac{1}{2}$ AC

$\Rightarrow$ ar (Δ APQ) = ar (Δ PQC) ……….(vi) [triangles between same parallel are equal in area]

From eq. (v) and (vi), we get

ar (Δ APQ) = ar (Δ ARC) ……….(vii)

R is the mid-point of AP. Therefore RQ is the median of Δ APQ.

$\therefore$ ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ APQ) ……….(viii)

From (vii) and (viii), we get,

ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ ARC)

(ii) PQ is the median of ΔBPC

$\therefore$ ar (ΔPQC) = $\frac{1}{2}$ ar (ΔBPC) = $\frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ……….(ix)

Also ar (ΔPRC) = $\frac{1}{2}$ ar (ΔAPC) [Using (iv)]

$\Rightarrow$ ar (ΔPRC) = $\frac{1}{2} \times \frac{1}{2}$ ar (ΔABC) = $\frac{1}{4}$ ar (ΔABC) ……….(x)

Adding eq. (ix) and (x), we get,

ar (ΔPQC) + ar (ΔPRC) = $\left(\frac{1}{4}+\frac{1}{4}\right)$ ar (ΔABC)

$\Rightarrow$ ar (quad. PQCR) = $\frac{1}{2}$ ar (ΔABC) ……….(xi)

Subtracting ar (ΔPRQ) from the both sides,

ar (quad. PQCR) – ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔABC) – ar (ΔPRQ)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{2}$ ar (ΔARC) [Using result (i)]

$\Rightarrow$ ar (ΔARC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{2} \times \frac{1}{2}$ ar (ΔAPC)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{4}$ ar (ΔAPC)

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{4} \times \frac{1}{2}$ ar (ΔABC) [PC is median of ΔABC]

$\Rightarrow$ ar (ΔRQC) = $\frac{1}{2}$ ar (ΔABC) – $\frac{1}{8}$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) = $\left(\frac{1}{2}-\frac{1}{8}\right) \times$ ar (ΔABC)

$\Rightarrow$ ar (ΔRQC) = $\frac{3}{8}$ ar (ΔABC)

(iii) ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔARC) [Using result (i)] $\Rightarrow$ 2 ar (ΔPRQ) = ar (ΔARC) ..(xii)

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔAPQ) [RQ is the median of ΔAPQ] ……….(xiii)

But ar (ΔAPQ) = ar (ΔPQC) [Using reason of eq. (vi)] ……….(xiv)

From eq. (xiii) and (xiv), we get,

ar (ΔPRQ) = $\frac{1}{2}$ ar (ΔPQC) ……….(xv)

But ar (ΔBPQ) = ar (ΔPQC) [PQ is the median of ΔBPC] ……….(xvi)

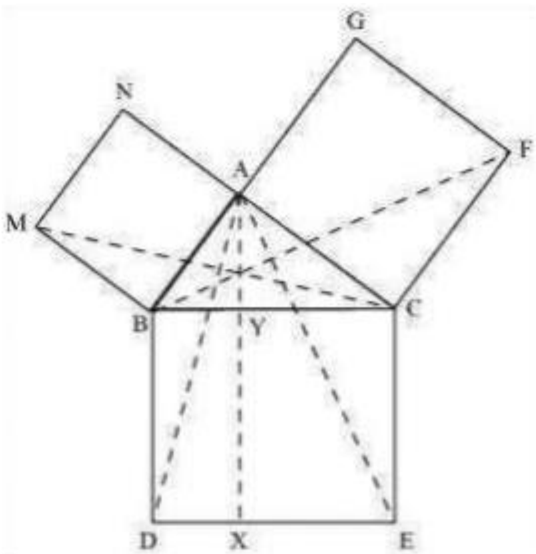
From eq. (xv) and (xvi), we get,

ar (Δ PRQ) = $\frac{1}{2}$ ar (Δ BPQ) ……….(xvii)

Now from (xii) and (xvii), we get,

$2\times\frac{1}{2}ar(\Delta BPQ)$ = ar (Δ ARC) $\Rightarrow$ ar (Δ BPQ) = ar (Δ ARC)

Question 8: In the following figure, ABC is a right triangle right angled at A. BCED, ACFG and ABMN are squares on the sides BC, CA and AB respectively. Line segment AX ⊥ DE meets BC at Y.



Show that:

(i) ΔMBC ≅ ΔABD
(ii) ar(BYXD) = 2ar(MBC)
(iii) ar(BYXD) = 2ar(ABMN)
(iv) ΔFCB ≅ ΔACE
(v) ar(CYXE) = 2ar(FCB)
(vi) ar(CYXE) = ar(ACFG)
(vii) ar(BCED) = ar(ABMN) + ar(ACFG)

Note: Result (vii) is the famous *Theorem of Pythagoras*. You shall learn a simpler proof of this theorem in class X.

Solution 8: (i) We know that each angle of a square is 90°.
Hence, ∠ABM = ∠DBC = 90º
∴ ∠ABM + ∠ABC = ∠DBC + ∠ABC
∴ ∠MBC = ∠ABD

In ΔMBC and ΔABD,
$\angle MBC = \angle ABD$ (Proved above)
MB = AB (Sides of square ABMN)
BC = BD (Sides of square BCED)
$\therefore \Delta MBC \cong \Delta ABD$ (SAS congruence rule)

(ii) We have
$\Delta MBC \cong \Delta ABD$
$\therefore$ ar (ΔMBC) = ar (ΔABD) ... (1)
It is given that $AX \perp DE$ and $BD \perp DE$ (Adjacent sides of square BDEC)
$\therefore BD \parallel AX$ (Two lines perpendicular to same line are parallel to each other)
ΔABD and parallelogram BYXD are on the same base BD and between the same parallels BD and AX.
Area (ΔYXD) = 2 Area (ΔMBC) [Using equation (1)] ... (2)
(iii) ΔMBC and parallelogram ABMN are lying on the same base MB and between same parallels MB and NC.
2 ar (ΔMBC) = ar (ABMN)
ar (ΔYXD) = ar (ABMN) [Using equation (2)] ... (3)
(iv) We know that each angle of a square is 90°.
$\therefore \angle FCA = \angle BCE = 90^\circ$
$\therefore \angle FCA + \angle ACB = \angle BCE + \angle ACB$
$\therefore \angle FCB = \angle ACE$
In ΔFCB and ΔACE,
$\angle FCB = \angle ACE$
FC = AC (Sides of square ACFG)
CB = CE (Sides of square BCED)
$\Delta FCB \cong \Delta ACE$ (SAS congruence rule)
(v) It is given that $AX \perp DE$ and $CE \perp DE$ (Adjacent sides of square BDEC)
Hence, $CE \parallel AX$ (Two lines perpendicular to the same line are parallel to each other)
Consider BACE and parallelogram CYXE
BACE and parallelogram CYXE are on the same base CE and between the same parallels CE and AX.
$\therefore$ ar (ΔYXE) = 2 ar (ΔACE) ... (4)
We had proved that
$\therefore \Delta FCB \cong \Delta ACE$
ar (ΔFCB) $\cong$ ar (ΔACE) ... (5)
On comparing equations (4) and (5), we obtain
ar (CYXE) = 2 ar (ΔFCB) ... (6)
(vi) Consider BFCB and parallelogram ACFG
BFCB and parallelogram ACFG are lying on the same base CF and between the same parallels CF and BG.

$\therefore$ ar (ACFG) = 2 ar (ΔFCB)

∴ ar (ACFG) = ar (CYXE) [Using equation (6)] ... (7)

(vii) From the figure, it is evident that

ar (ΔCED) = ar (ΔYXD) + ar (CYXE)

∴ ar (ΔCED) = ar (ABMN) + ar (ACFG) [Using equations (3) and (7)].